Energy budget-based backscatter in a shallow water model driven by double gyre wind forcing

Milan Klöwer1,2,*, Richard J Greatbatch1,2, Sören Thomsen3, Martin Claus1, and Malte F Jansen1

1GEOMAR Helmholtz Centre for Ocean Research Kiel, Kiel, Germany
2University of Kiel, Kiel, Germany
3Department of the Geophysical Sciences, University of Chicago, Chicago, USA
*Corresponding author: milankloewer@gmx.de

Abstract
The parametrization of sub-grid scale processes is one of the key challenges towards improved numerical simulations of atmospheric and oceanic circulation. Numerical weather prediction models as well as climate models would benefit from more sophisticated turbulence closures that allow for less spurious dissipation at the grid-scale and consequently higher and more realistic levels of eddy kinetic energy (EKE). Recent studies1,2 propose to use a hyperviscous closure in combination with an additional deterministic forcing term acting as negative viscosity to represent backscatter of energy from the unresolved scales. Here, we apply the parametrization to a shallow water model driven by double gyre wind forcing with no-slip boundary conditions and provide evidence for its general application.

Formulation: Shallow water model with backscatter
The sub-grid EKE is introduced as an additional prognostic variable e, that is fed by dissipation at the grid scale, and enables recycling of EKE via the backscatter term (ξx, ξy) at larger scales

\[
\begin{align*}
\theta u + u \partial_x u + v \partial_y u - f v &= -g \partial_x \eta + \frac{F_x}{\rho h} + M_x + \epsilon_x \\
\theta v + u \partial_x v + v \partial_y v + f u &= -g \partial_y \eta + M_y + \epsilon_y \\
\partial_t \eta + \partial_x (uh) + \partial_y (vh) &= 0
\end{align*}
\]

with lateral mixing of momentum with a biharmonic stress tensor \( S^* \)

\[
(M_x, M_y) = \nu_B h^{-1} \nabla \cdot h S^* \approx \nu_B \nabla^2 u
\]

and the dissipated, backscattered energies

\[
\begin{align*}
\dot{E}_{\text{diss}} &= c_{\text{diss}} \nu_B h \nabla u \cdot S^* \\
\dot{E}_{\text{back}} &= \nu_{\text{back}} \nabla^2 u \cdot S
\end{align*}
\]

Backscatter terms as Laplacian viscosity

\[
(\xi_x, \xi_y) = \nabla^{-1} \nabla \cdot \nu_{\text{back}} h S
\]

Energy budget-based backscatter schematically: The energy being dissipated by biharmonic viscosity is partly captured in the sub-grid EKE (green), and partly removed where \( c_{\text{diss}} < 1 \). Based on the local levels of sub-grid EKE a negative Laplacian viscosity re-injects energy at larger scales to recycle some of the spurious dissipation at the grid-scale.

Improvements on the mean state

Improvements on variability

Sub-grid eddy kinetic energy

Discussion
The energy-budget based backscatter parametrization effectively lowers the viscosity but keeps the numerical model stable by an artificial upscale transfer of energy, that results from the combination of biharmonic diffusion (i.e. the lateral mixing of momentum term) and harmonic anti-diffusion (i.e. backscatter term). The parametrization tends to accelerate eddies, that would otherwise be dissipated. We realize that increasing the amount of kinetic energy in the low resolution model compared to the high resolution truth yields a better simulation with respect to a variety of statistical measures but also pushes the model towards the edge of numerical stability. In a shallow water model with bottom friction the improvements of the backscatter formulation are assumed to be even greater, due to lowered levels of kinetic energy.

Using the backscatter formulation as presented here increases the computational cost by a factor of 1.5 compared to a factor of at least 16 by quadrupling the resolution. A wider application of energy-budget based backscatcer is therefore promising in order to remove model biases as well as increasing the quality of predictions.

References

Model code written in Python

github.com/milankl/swm